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Magnetic monopoles in 4D: a perturbative calculation

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Abstract: We address the question of defining the second quantised monopole creation operator in the 3+1 dimensional Georgi-Glashow model, and calculating its expectation value in the confining phase. Our calculation is performed directly in the continuum theory within the framework of perturbation theory. We find that, although it is possible to define the "coherent state" operator $M(x)$ that creates the Coulomb magnetic field, the dependence of this operator on the Dirac string does not disappear even in the nonabelian theory. This is due to the presence of the charged fields ($W^{\pm}$). We also set up the calculation of the expectation value of this operator in the confining phase and show that it is not singular along the Dirac string. We find that in the leading order of the perturbation theory the VEV vanishes as a power of the volume of the system. This is in accordance with our naive expectation. We expect that nonperturbative effects will introduce an effective infrared cutoff on the calculation making the VEV finite.

Contents

1. Introduction

Common lore has it that the condensation of magnetic monopoles is responsible for confinement in nonabelian gauge theories [1]. Numerous lattice investigations of the monopoles in pure Yang-Mills theories are available in the literature [2]. Nevertheless, there are many open questions regarding the status of magnetic monopoles in confining theories. The main problem with the monopole condensation scenario is that no gauge invariant definition of the monopole operator can be given in most theories of interest. Moreover, no gauge invariant observable corresponding to magnetic charge exists at least in the SU(2) pure Yang-Mills theory [3]. Thus serious doubts remain regarding the significance of the monopoles for confinement in pure gluodynamics.

On the other hand there undoubtedly exist theories where magnetic monopoles can be given a proper gauge invariant meaning. A prime example of such a theory is the Georgi-Glashow model in 3+1 dimensions. The model comprises of the SU(2) gauge field coupled to the adjoint Higgs

$$L = -\frac{1}{4} F^2_{\mu\nu} + (D_\mu H)^2 - V(H^2)\,. \tag{1.1}$$

The theory is believed to have two phases as a function of the Higgs potential $V(H^2)$. In the Higgs phase, where the VEV of the Higgs field is nonvanishing, the magnetic monopoles exist in the spectrum as heavy particles. Closer to the boundary of the confining phase they become light, and it is natural to expect that they condense in the confining phase of the theory. The magnetic charge (which, as opposed to the pure Yang Mills theory, does exist here as a physical observable) is then expected to be spontaneously broken in the confining phase.

This behaviour has been all but established in the supersymmetric cousin of the Georgi-Glashow model [4]. It is however worth noting, that the confinement in the deformed N=2 supersymmetric theory is essentially abelian, and thus differs from the confinement in QCD in many important aspects [5]. Confinement in the Georgi-Glashow model on the other hand is expected to be fully nonabelian, and thus bear close similarity to QCD.

Since the magnetic charge certainly exists as a physical observable in the Hilbert space of the Georgi-Glashow model, one is tempted to try to use this model as crutches on the way to the pure Yang-Mills theory. More specifically, one would like to start with the Georgi-Glashow model and trace the fate of the monopoles when the mass of the Higgs field in the confining phase is taken to be very large. In this limit the theory becomes the pure SU(2) Yang-Mills theory. Of course, the perturbation theory, which is valid in the Higgs phase, is not valid deep in the confining regime anymore. Nevertheless, one can hope that the heavy Higgs field can be integrated out perturbatively so that one can at least understand what becomes of the monopole creation operator in the pure Yang-Mills limit. Since we know that the magnetic charge disappears in this limit, two options seem to be open for the monopole creation operator. The first one is that it simply reduces to some run-of-the-mill operator with the vacuum quantum numbers. This operator could conceivably be a convenient order parameter for confinement, even though it does not probe symmetry breaking any more. The other possibility is that the condensate of the Georgi-Glashow monopoles deep in the confining phase is so stiff, that it costs the energy of the order of the Higgs mass to excite it. In this case any excitation coupled to the condensate will be very heavy, and in the limit the monopole creation operator would simply reduce to a unit operator. In this case, it would become useless for discussions of the properties of pure Yang-Mills theory. In particular it could not be an order parameter for the deconfining phase transition at finite temperature. The former option seems more likely, since generically one expects light scalar excitations (glueballs) to couple to the "radial" part of the magnetic monopole field, and thus to excite the condensate. Nevertheless the second option is a logical possibility worth exploring/ruling out.

To study this question one has to start from the beginning, and the beginning appears to be the construction of the monopole creation operator in the continuum Georgi-Glashow model. Surprisingly, although there exist a large number of lattice simulations in the pure Yang-Mills theory, there have been to our knowledge no attempts to explore this aspect of the Georgi-Glashow model. The continuum literature is also sorely missing on this point. There have been attempts to construct the monopole operator in the Georgi-Glashow model [6], but they appear to be incomplete. The aim of the present paper is to construct a gauge invariant monopole creation operator in the Georgi-Glashow model, and to set up the perturbative calculation of its expectation value in the confining phase. We construct a continuum gauge invariant operator which creates the Coulombic magnetic field (without the Dirac string) of the monopole and has the correct commutation relation with the magnetic charge density:

$$
\begin{aligned}
[M(x), B_k(y)] &= \frac{i}{g}\,\frac{(x-y)_k}{(x-y)^3}\,M(x)\,, \\
[M(x), j_0^M(y)] &= \frac{4\pi}{g}\,\delta^3(x-y)\,M(x)\,.
\end{aligned}
\tag{1.2}
$$

This operator is constructed by analogy with lower dimensional theories as a "coherent state" creation operator, that is an exponential of an operator linear in electric fields. An important property of the monopole operator is that it is not rotationally invariant even

though the magnetic field created by it is spherically symmetric. This is directly related to the fact that the Dirac string is present in the definition of the monopole operator. The direction of the string has to be specified and it affects the commutators of M with local gauge invariant quantities, like the spacial components of electric current. We stress that these commutators are not hopelessly divergent at the position of the string, but merely depend on its orientation.

We are also able to set up the perturbative calculation of the expectation value of M in the confining phase. We find that, as in the calculation of similar averages in lower dimensions [7], the relevant path integral is dominated by a classical configuration. This configuration interpolates in time between two states with magnetic charge one half (in units of the magnetic charge of the 'tHooft-Polyakov monopole). At time plus infinity the magnetic charge density is spread out homogeneously over space, while at minus infinity the state contains a point-like 'tHooft-Polyakov monopole and a balancing charge minus one half spread out homogeneously in space. Viewed from a four dimensional perspective, the configuration looks like radial magnetic current emanating from the endpoint of a stem (world line of the 'tHooft-Polyakov monopole). We therefore call this configuration "a dandelion". The physical amplitude described by this configuration corresponds to the off diagonal matrix element between two components of the vacuum wave function $\langle\Psi_1|M|\Psi_0\rangle$, where $|\Psi_1\rangle$ is a state with magnetic charge one, and $|\Psi_0\rangle$ a state with magnetic charge zero. This is a typical situation in theories with a spontaneous broken symmetry (in the present case the symmetry generator being the magnetic charge). A surprising property of the leading dandelion configuration is that it is not spherically symmetric as one might naively expect. This is the direct consequence of the lack of rotational invariance in the definition of the monopole operator. The problem of string dependence is not new and is well known in the framework of the lattice gauge theory [8]. A possible solution to it has been suggested by Fröhlich and Marchetti [9]. Although the prescription of [9] directly applies to the calculation of the correlation functions, and not to the operator itself, roughly speaking it corresponds to the definition of M as a weighted average over operators which create two Z_2 strings rather than a Coulomb magnetic field. This prescription goes outside the framework of the coherent state operator, which we have adopted here and is much more complicated. We therefore do not consider it in this paper, and we believe that it will not alter qualitatively our results.

Although the structure of the leading configuration suggests spontaneous breaking of magnetic charge, we still find $\langle M\rangle = 0$ even in the confining phase. The reason for this is that the classical action of the dandelion is infrared divergent. In particular, as a function of the infrared cutoff (the size of the system V) we find $\langle M\rangle \propto \exp\{-\frac{\pi c}{g^2}\ln V\}$, where c is a pure number of order unity. This is not unexpected for the simple reason that in the perturbation theory the gluons are massless. In the presence of massless particles the theory has no scale, and thus we expect a logarithmic divergence of the classical action. The situation in the perturbation theory is thus similar to the Kosterlitz-Thouless type phases in 1+1 dimensional theories [10], where continuous symmetries are "almost" spontaneously broken and correlators of the order parameter decay not exponentially, but rather as a power of the distance. Beyond perturbation theory in the Georgi-Glashow model we expect the situation

to change. Nonperturbative effects will introduce an infrared scale, the confinement radius, and we expect this scale to cut off the infrared divergence of the classical action and to lead to a finite VEV of M.

This paper is organised as follows. In Section 2 we discuss the construction of the monopole operator. In Section 3 we set up the calculation of its expectation value and discuss the properties of the dandelion configuration. Section 4 contains some brief comments.

2. The monopole creation operator

The classical field configuration of the 'tHooft-Polyakov monopole is

$$H^a = h(r)\frac{r^a}{r}\,, \quad A^a_i = \frac{f(r)}{g}\epsilon_{aib}\,\frac{r^b}{r^2}\,, \tag{2.1}$$

where $h(0) = f(0) = 0$, $\lim_{r\to\infty} h(r) = v$, $\lim_{r\to\infty} f(r) = 1$ and v is the vacuum expectation value of the Higgs field. The distance scale on which the fields approach their asymptotic values is given by their masses. Since we are only interested in the infrared properties of the theory, we will look for an operator which creates a point-like monopole and thus from this point on fix $h(r) = v$, $f(r) = 1$.

This configuration carries one unit of magnetic charge. The gauge invariant magnetic charge in the Georgi-Glashow model can be defined in the following way [1]. First one defines a gauge invariant field strength as

$$\mathrm{F}_{\mu\nu}(x) = \frac{H^a}{|H|}F^a_{\mu\nu} - \frac{1}{g}\,\frac{1}{|H|^3}\,\epsilon^{abc}\,H^a\,(D_\mu H)^b\,(D_\nu H)^c\,, \tag{2.2}$$

in terms of the nonabelian field strength tensor

$$F^a_{\mu\nu} = \partial_\mu\,A^a_\nu - \partial_\nu\,A^a_\mu + g\,\epsilon^{abc}A^b_\mu\,A^c_\nu\,,$$

and covariant derivatives of the Higgs field H^a

$$(D_\mu H)^a = \partial_\mu\,H^a + g\,\epsilon^{abc}A^b_\mu\,H^c\,.$$

The 'tHooft tensor (2.2) can also be written as

$$\mathrm{F}_{\mu\nu} = \partial_\mu\left(\hat{H}^a\,A^a_\nu\right) - \partial_\nu\left(\hat{H}^a\,A^a_\mu\right) - \frac{1}{g}\,\epsilon^{abc}\,\hat{H}^a\partial_\mu\hat{H}^b\partial_\nu\hat{H}^c\,, \tag{2.3}$$

with the unit vector field $\hat{H}^a = H^a/|H|$. The magnetic current is then defined as

$$k^M_\mu(x) = \frac{1}{2}\,\epsilon_{\mu\nu\lambda\sigma}\,\partial^\nu\mathrm{F}^{\lambda\sigma}(x)\,. \tag{2.4}$$

This definition is convenient since the magnetic charge density so defined for all smooth configurations is equal to the topological charge density of the Higgs field. The total magnetic charge is then equal to the topological charge of $\hat{H}$.

$$Q_M = \frac{1}{4\pi}\int d^3x\,k^M_0\,(x) = \frac{1}{8\pi\,g}\int \mathrm{d}^2S_i\,\epsilon_{ij\,k}\epsilon^{abc}\hat{H}^a\partial_j\hat{H}^b\partial_k\hat{H}^c\,, \tag{2.5}$$

where the last integral is over the surface surrounding the zeros of the Higgs field. It should be noted that the definition eq. (2.4) is not unique. One can add to this current any other conserved current with the same quantum numbers whose charge vanishes on the 'tHooft-Polyakov monopole configuration. In particular it is useful to consider

$$j_\mu^M = \epsilon_{\mu\nu\lambda\sigma}\partial^\nu \left(\hat{H}^a(x) F^{\lambda\sigma\, a}(x) \right) . \tag{2.6}$$

On the 'tHooft-Polyakov monopole configuration the charge corresponding to this current is the same as Q_M since the covariant derivative of the Higgs field vanishes outside the monopole core. On the other hand the density of this charge, unlike that of eq. (2.5), is not constrained to vanish outside the points where $H = 0$, and thus behaves like the majority of other local operators. For this reason we prefer to use this alternative definition of the magnetic current in this paper.

Our aim is to construct a quantum operator that creates a monopole in the fully second quantised theory. This operator has to satisfy the following commutation relations

$$[M(x),\, j_0^M(y)] = \frac{4\pi}{g}\, \delta^3(x-y)\, M(x) . \tag{2.7}$$

It is natural to constrain the operator M further by requiring a somewhat more restrictive commutator

$$M(x) B_k(y) M(x)^\dagger = B_k(y) + \frac{i}{g} \frac{(x-y)_k}{(x-y)^2} , \tag{2.8}$$

where $B_k = \frac{1}{2}\epsilon_{klm}\hat{H}^a F^a_{lm}$. This latter commutator ensures that the operator M creates a Coulomb like magnetic field.

These commutation relations are reminiscent of the ones satisfied by soliton creation operators in 2 and 3 dimensions [7]. There is however one crucial difference, and that is that the monopole is a nonlocal object, and thus has a nonlocal commutation relation with local gauge invariant fields, e.g. $B_i(x)$.

The most natural way of going about the construction of M is to try a coherent state like operator, that is an exponential of the linear functional of electric fields. We thus start with the following ansatz

$$M(x) = D(x) M_A(x) , \tag{2.9}$$

where

$$M_A(x) = \exp\left(i \int \mathrm{d}^3 y\, \lambda_i(x-y) \hat{H}^a(y) E_i^a(y) \right) , \tag{2.10}$$

with λ_i the classical vector potential of a point-like Dirac monopole:

$$\lambda_i(x) = \frac{1}{g}\, \epsilon_{ij}\, \frac{r_\perp^j}{r_\perp^2}\, (\cos\theta - 1) . \tag{2.11}$$

Here we have chosen the Dirac string to run in the direction of the negative x_3 axis, and have defined $\mathbf{r}_\perp = (x_1, x_2)$. The operator M_A creates a vector potential of the Dirac monopole which in the isospace is oriented in the direction of the Higgs field. In particular

$$M_A(x) B_i(y) M_A^\dagger(x) = B_i(y) + \frac{i}{g}\left[\frac{(x-y)_i}{(x-y)^2} - 4\pi\, \hat{z}_i\, \delta^2(x_\perp - y_\perp)\theta(-(x_3-y_3)) \right] . \tag{2.12}$$

Note that the operator M_A is fully gauge invariant. It does however create the magnetic field of the Dirac monopole including the contribution of the Dirac string. As a result it does not create a magnetic charge, since the magnetic flux due to the Coulomb magnetic field is balanced by the flux of the Dirac string. The role of the operator D in eq. (2.9) is to create the magnetic field of the negative Dirac string alone, and thus cancel the unwanted singular magnetic field in eq. (2.12).

To construct the string operator D we turn for inspiration to the classical monopole solution eq. (2.1). This solution is written in the gauge where the vector potential is regular everywhere except at the location of the monopole. One can however transform this into unitary gauge, where the Higgs field is constant everywhere in space. The transformation that achieves this is given by

$$U(x) = \exp\{-i\frac{\theta}{2}\,\sigma\cdot\hat{\phi}\}\,, \tag{2.13}$$

where θ and ϕ are azimuthal and polar angles respectively, σ are Pauli matrices and the unit vector

$$\hat{\phi}^i = |x_\perp|\partial_i\phi = -\epsilon_{ij}\frac{x_\perp^j}{|x_\perp|}\,. \tag{2.14}$$

The formal gauge transformation with this gauge matrix gives

$$\begin{aligned} U^\dagger(x)A_i(x)U(x) \,+\, \frac{2i}{g}\,U^\dagger(x)\partial_i U(x) &= \lambda_i(x)\sigma_3\,, \\ U^\dagger(x)\hat{H}(x)U(x) &= \sigma_3\,, \end{aligned} \tag{2.15}$$

where we have defined $A_i \equiv A_i^a\sigma^a$ and $\hat{H} \equiv \hat{H}^a\sigma^a$. The transformation eq. (2.15) is not a proper gauge transformation. The rotation matrix U is singular along the negative x_3 axis, where the direction of rotation $\hat{\phi}$ is undefined. As a result the transformed configuration is not equivalent to eq. (2.1), as the magnetic field corresponding to eq. (2.15) has an extra Dirac string, and the magnetic charge of eq. (2.1) is cancelled by the string contribution. The second quantised version of the transformation eq. (2.13) (or rather its inverse) is precisely what we need to define to be able to get rid of the Dirac string contribution in eq. (2.9). It is however somewhat cumbersome to work directly with the transformation eq. (2.13) as it changes the gauge variant variables everywhere in space. It is more convenient to redefine it by a proper gauge transformation which achieves the same result as eq. (2.13) everywhere except infinitesimally close to the Dirac string.

Define

$$\tilde{U}(x) = \exp\{-i\frac{\tilde{\theta}}{2}\,\sigma\cdot\hat{\phi}\}\,, \tag{2.16}$$

where the function $\tilde{\theta}$ is equal to the azimuthal angle θ everywhere except inside an infinitely narrow tube around the negative x_3 axis, e.g.

$$\tilde{\theta} = \theta\left[1 - \frac{1}{2}(1-\mathrm{sign}(x_3))f(x_\perp)\right], \qquad f(0)=1, \qquad f(x>\epsilon)=0\,. \tag{2.17}$$

The action of this transformation on the configuration eq. (2.1) gives

$$
\begin{aligned}
\tilde{H} &\equiv \tilde{U}^\dagger \hat{H} \tilde{U} = \sigma_3 \cos(\theta - \tilde{\theta}) + \sigma \cdot \hat{r}_\perp \sin(\theta - \tilde{\theta})\,, \\
\tilde{A}_i &\equiv \tilde{U}^\dagger(x) A_i(x) \tilde{U}(x) + \frac{2i}{g}\, \tilde{U}^\dagger(x) \partial_i \tilde{U}(x) \\
&= -\frac{1}{g}\Big(\partial_i\phi [\cos(\theta - \tilde{\theta}) - \cos\theta] \tilde{H} - \partial_i(\tilde{\theta} - \theta) P_1 - \partial_i\phi \sin(\theta - \tilde{\theta}) P_2 \Big)\,,
\end{aligned}
\tag{2.18}
$$

where

$$
P_1 = \sigma \cdot \hat{\phi}\,, \qquad P_2 = \cos(\theta - \tilde{\theta})\, \sigma \cdot \hat{r}_\perp - \sin(\theta - \tilde{\theta})\, \sigma_3\,. \tag{2.19}
$$

In the formal limit $\tilde{\theta} = \theta$ the configuration eq. (2.18) becomes equivalent to eq. (2.15), however at any nonzero value of the regulator ϵ in eq. (2.17) it is fully gauge equivalent to eq. (2.1).

The transformation

$$
V(x) = \tilde{U}^\dagger(x) U(x) = \exp\{-i\frac{\theta - \tilde{\theta}}{2}\, \sigma \cdot \hat{\phi}\} \tag{2.20}
$$

is gauge equivalent to $U(x)$, but acts even on gauge variant fields only in the infinitesimal neighbourhood of the negative x_3 axis. To construct the operator that creates the Dirac string we must understand how to write down the second quantised version of the transformation eq. (2.20). The difficulty here is in the fact that in writing down the matrix V we have assumed explicitly that the field $\hat{H}$ on which it is acting points in the third direction in the isospace. For any other background, V should be additionally rotated by a regular, field dependent transformation. It is however not very transparent how to deal with a transformation whose parameters themselves depend on quantum fields. The alternative is to take the limit $\epsilon \to 0$, in which case the Higgs field does not change at all under the action of the monopole operator. The vector potential however is affected by the transformation V even in this limit. Clearly, if $\hat{H}$ does not change, the vector potential classically must acquire a non singlevalued piece if the resulting magnetic field is to have a nonvanishing divergence. It is not difficult to write down a multivalued vector potential which gives a magnetic field of a Dirac string.

$$
\mu_3^a = \mu_\phi^a = 0, \qquad \mu_r^a = \frac{1}{g}\, \phi\, \delta(r) \theta(-x_3)\, \hat{H}^a\,, \tag{2.21}
$$

where the radial delta function is one dimensional rather than two dimensional to preserve the correct dimensionality of the vector potential. A shift of A by μ is affected by the following second quantised operator

$$
D(x) = \exp\left(\frac{i}{g} \int \mathrm{d}^3y\, \phi(x-y)\, \delta(|\mathbf{x}_\perp - \mathbf{y}_\perp|) \frac{(x_\perp - y_\perp)_i}{|\mathbf{x}_\perp - \mathbf{y}_\perp|}\, \theta(-(x_3 - y_3))\, \hat{H}^a(y) E_i^a(y) \right). \tag{2.22}
$$

Even though this expression looks suspect, it is in fact a well defined operator in a nonabelian theory. On the microscopic level (ultraviolet cutoff scale) the electric field operator in a nonabelian theory has a discrete spectrum which is quantised in units of g. Thus the

apparent ambiguity in the definition of ϕ is not felt by any physical observable since it only leads to ambiguity of an integer multiple of 2π in the phase of eq. (2.22). The operator $D(x)$ is gauge invariant and is similar in form to $M_A(x)$. In the following therefore we will not write it down explicitly. We will continue denoting the vector potential of the monopole by λ_i but will understand it as the sum of λ_i of eq. (2.11) and μ_i of eq. (2.21).

Hence, with this definition the monopole creation operator satisfies the correct commutation relation with the abelian magnetic field, eq. (2.8). This commutator is independent of the direction of the Dirac string, even though the definition of M uses explicitly the Dirac string in a particular direction. However the dependence on the string does not disappear from all commutators of M with gauge invariant operators. In particular consider the following commutator

$$\begin{aligned} M_A^\dagger(x)\left[(B_i^a(y))^2 - B_i^2(y)\right] M_A(x) = & \left[(B_i^a(y))^2 - B_i^2(y)\right] + 2\epsilon_{ijk} B_i^a(y)\lambda_j(y-x)(D_k\hat{H}(y))^a \\ & + \left[\epsilon_{ijk}\lambda_j(y-x)(D_k\hat{H}(y))^a\right]^2 . \end{aligned} \tag{2.23}$$

This explicitly depends on the vector potential λ_i.

We note that the commutator eq. (2.23) has a direct analog in the abelian theory. Consider an abelian gauge theory with a charged scalar field ϕ. The action of an abelian monopole creation operator on the spatial components of electric current $\phi^* D_i \phi$ is

$$M^\dagger(x)\left[\phi^* D_i\phi(y) - D_i\phi^*\phi(y)\right] M(x) = \left[\phi^* D_i\phi(y) - D_i\phi^*\phi(y)\right] + i\lambda_i(x-y)\phi^*(y)\phi(y) . \tag{2.24}$$

Thus the action of the monopole creation operator produces a long range electric current. Similarly the gauge coupled kinetic term of the scalar field transforms as

$$M^\dagger(x) D_i\phi^* D_i\phi(y) M(x) = D_i\phi^* D_i\phi(y) - i\lambda_i(x-y)\left(\phi^* D_i\phi(y) - D_i\phi^*\phi(y)\right) + \lambda_i^2\phi^*\phi . \tag{2.25}$$

It is this property of the magnetic monopole operator in theories with charged matter fields that leads to dependence of the operator on the choice of the string, as has been discussed extensively in [8, 9].

The commutator in eq. (2.23) is very similar in physical meaning. The operator $D_i\hat{H}$ is naturally identified with the charged vector gauge boson fields $W^\pm$ (the two components of the vector field perpendicular to the Higgs). The square of the nonabelian magnetic field contains the minimal coupling of the charged vector field $W^\pm$ to the abelian gauge potential. Thus eq. (2.23) is simply a transcription of eq. (2.25) in terms of spin one charged matter. The potentially disturbing property of the commutator eq. (2.23) is that it is much more nonlocal than the commutator with the abelian magnetic field. Whereas the abelian magnetic field created by the monopole operator decreases as $1/(x-y)^2$, the nonabelian magnetic field decreases only as $1/|x-y|$. As we shall see in the next section this nonlocality has an important effect on the calculation of the expectation value of the monopole creation operator in the confining phase.

3. Perturbative calculation of $\langle M \rangle$

In this section we set up the calculation of the expectation value of the monopole cre-

ation operator in the confining phase. Although we do not expect the perturbative result to be reliable, we do expect to see substantial differences between the behaviour of the expectation value in the confining and Higgs phases.

In the Higgs phase, since the monopoles are massive particles, the correlation function of the monopole creation operator at large distance should behave as

$$\langle M^*(x)M(y)\rangle \propto \exp\left(-\mu|x-y|\right), \tag{3.1}$$

where μ is the monopole mass. Thus the expectation value in the finite but large volume is

$$\langle M\rangle \propto \exp\left(-\mu L\right). \tag{3.2}$$

In the confining phase we expect the operator M to have a finite expectation value. However, since confinement is intrinsically nonperturbative, we still expect to find a vanishing expectation value in the perturbative setup. However we do expect the volume dependence at finite volume to be much milder.

Consider the path integral calculation of $\langle M\rangle$:

$$\langle M\rangle = \int D\hat{H}DA\exp\left(-S[A]\right)M[A] \sim \int DA\exp\left(-\frac{1}{4}\int \mathrm{d}^4x\,(F^a_{\mu\nu}-f^a_{\mu\nu})^2\right), \tag{3.3}$$

with

$$f^a_{0i} = \lambda_i(x)\hat{H}^a\,\delta(x_4)\,, \qquad f_{ij}=0\,. \tag{3.4}$$

We are interested in calculating the expectation value deep in the confining phase, where the Higgs field is very heavy. In the first approximation we therefore neglect the Higgs contribution to the action. The c-number field eq. (3.4) is the shifted Dirac vector potential, which multiplies the operator of the electric field in the exponent eq. (2.10). The extra c-number contribution to the action f^2 arises when writing the expectation value in the functional integral form as discussed in detail for example in [11].

Path integrals of this type arise frequently in calculating expectation value of operators which create topological excitations [7]. At weak coupling the path integral can be calculated in the steepest descent approximation. This becomes obvious once one realises that a simple rescaling of fields $A\to\frac{1}{g}A$, leads to the appearance of the factor $1/g^2$ in front of the action in eq. (3.3).

Our aim is therefore to find a classical configuration which minimises the action in eq. (3.3). First we cast the path integral expression into a more intuitively appealing form by changing variables

$$A^a_\mu(x)\to A^a_\mu(x)-\lambda_\mu(\vec{x})\theta(-x_4)\hat{H}^a\,, \tag{3.5}$$

with $\lambda_0=0$ and the spatial components of λ_i given by the sum of eq. (2.11) and eq. (2.21).

Under this transformation

$$F^a_{\mu\nu}\to F^a_{\mu\nu}-\left(\partial_{[\mu}\lambda_{\nu]}\hat{H}^a+\lambda_{[\nu}(D_{\mu]}\hat{H})^a\right)\theta(-x_4)-\lambda_{[\nu}\,\delta_{\mu]4}\,\hat{H}^a\,\delta(x_4)\,. \tag{3.6}$$

The path integral for the calculation of VEV of the monopole operator becomes

$$\langle M\rangle = \int DA\exp\left\{-\frac{1}{4}\left(F^a_{\mu\nu}-\bar{f}_{\mu\nu}\,\hat{H}^a-\left(\lambda_\nu D_\mu\hat{H}-\lambda_\mu D_\nu\hat{H}\right)^a\theta(-x_4)\right)^2\right\}, \tag{3.7}$$

where $\bar{f}_{\mu\nu}$ is the magnetic field of a point-like 'tHooft-Polyakov monopole at $x_4 < 0$,

$$\bar{f}_{ij} = \frac{1}{g}\,\epsilon_{ijk}\,\frac{x_k}{x^3}\,\theta(-x_4)\,. \tag{3.8}$$

The general properties of the steepest descent configuration that minimises the action in eq. (3.7) can be understood using the following argument. The insertion of the operator M into the path integral annihilates a magnetic monopole at time $t = 0$. Thus one expects that the leading configuration has a smooth magnetic charge density which at negative times integrates to a different value than at positive times, the difference being the magnetic charge of the monopole annihilated by M. One might expect that the action is minimised by a spherically symmetric configuration. The natural candidate for such a magnetic charge/current density would be

$$j_\mu(x) = \frac{1}{g}\,\frac{x_\mu}{|x|^4}\,, \tag{3.9}$$

since it has a divergence at the point in space-time where the monopole is created, and thus is naturally associated with a space time event of annihilation of a monopole. The overall coefficient is determined by the requirement that the 3D flux emanating from this point is the same as that created by M. Such a current cannot however be constructed from the field strength at all, since by definition any magnetic current that can be written in terms of the fundamental fields of the theory is divergeneseless. Thus one needs to balance the total flux that emanates from the point $x = 0$. The simplest and the most natural way to do that is to have the balancing flux going in an infinitely thin flux tube:

$$j^M_\mu = \frac{1}{g}\left[\frac{x_\mu}{|x|^4} - 4\pi\,\delta_{\mu 4}\,\delta^3(\vec{x})\,\theta(-x_4)\right]\,. \tag{3.10}$$

The thin flux tube term in eq. (3.10) has a natural interpretation of the world line of the monopole annihilated by the operator M. This structure is very similar to the one arising in the calculation of the VEV of the magnetic vortex operator in 2+1 dimensions [7] and is generic for a broken symmetry phase. Pictorially it represents the flux of the spontaneously broken charge emanating from a single space-time point in a spherically symmetric way, like the head of a dandelion, while the necessary "nutrients" (incoming flux) are supplied by a thin stem extending infinitely into the past. We will thus refer to a configuration of this generic type as a "dandelion".

We will find however that a spherically symmetric dandelion does not minimise the classical action. Instead the action is lower for an axially symmetric dandelion, the spherical symmetry being broken due to the sensitivity to the direction of the Dirac string. In the rest of this section we will construct the vector potential for an axially symmetric dandelion and consider the minimisation of the action. Although we cannot prove that the configurations of the type we consider lead to an absolute minimum of the action, we believe that parametrically (and physically) our results are correct.

We start with the following expression for an axially symmetric vector potential and the Higgs field in four Euclidean dimensions:

$$\hat{H}^a = \frac{x^a}{r}\,, \qquad A^a_i = \frac{f(z,x)}{g}\,\epsilon_{aib}\frac{x^b}{r^2}\,, \tag{3.11}$$

where

$$r^2 = x_1^2 + x_2^2 + x_3^2, \qquad z = \frac{x_4}{r}\,, \qquad x = \cos\theta\,. \tag{3.12}$$

Calculating the components of the field strength we find

$$F^a_{ij} = F_{ij}\hat{H}^a + \frac{1}{g}\frac{x_k}{r^3}\left[\epsilon^{ajk}\left(-\frac{x_i}{r}\,z\frac{\partial f}{\partial z} + (\delta_{i3} - \frac{x_3 x_i}{r})\,\frac{\partial f}{\partial x}\right) - \;(i \leftrightarrow j)\;\right]\,, \tag{3.13}$$

$$F^a_{4i} = \partial_4 A^a_i = \frac{1}{g}\epsilon_{aib}\frac{x^b}{r^2}\partial_4 f(z,x) = \frac{1}{g}\,\epsilon_{aik}\frac{x_k}{r^3}\,\frac{\partial f}{\partial z}\,, \tag{3.14}$$

where

$$F_{ij} = -\frac{1}{g}\frac{\epsilon_{ijk}x_k}{r^3}\,(2f - f^2)\,. \tag{3.15}$$

The covariant derivative of the Higgs field is given by

$$(D_i H)^a = \frac{1}{r}\,(\delta^{ai} - \frac{x_i x_a}{r^2})(1-f)\,. \tag{3.16}$$

The magnetic current for this configuration is

$$j^M_\mu = \frac{1}{g}\,\left[\frac{x_\mu}{r^4}(1-f)\frac{\partial f}{\partial z} - 4\pi\delta_{\mu 4}\,\delta^3(\vec{x})\theta(-x_4)\right]\,. \tag{3.17}$$

This looks like a radial dandelion but with the density of the current not spherically symmetric - a dandelion in the wind.

With these expressions we find that the classical action in eq. (3.7) is given by the following integral

$$S_{\text{dandelion}} = -\frac{\pi}{g^2}\int_0^\infty \frac{dr}{r}\,[S_1 + S_2 + S_3 + S_4 + S_5]\,, \tag{3.18}$$

with

$$\begin{aligned}
S_1 &= \int_{-\infty}^{\infty} dz \int_{-1}^{1} dx\,\left[2f - f^2 - \theta(-z)\right]^2\,, \qquad (3.19)\\
S_2 &= \int_{-\infty}^{\infty} dz \int_{-1}^{1} dx\,\left(\frac{\partial f}{\partial z}\right)^2\,,\\
S_3 &= \int_{-\infty}^{\infty} dz \int_{-1}^{1} dx\,2z^2\left(\frac{\partial f}{\partial z}\right)^2\,,\\
S_4 &= \int_{-\infty}^{\infty} dz \int_{-1}^{1} dx(1-x^2)\left(\frac{\partial f}{\partial x}\right)^2\,,\\
S_5 &= \int_{-\infty}^{\infty} dz \int_{-1}^{1} dx\,\theta(-z)(1-x)(1-f)\left[\frac{1}{1+x}(1-f) - 2\frac{\partial f}{\partial x}\right]\,.
\end{aligned}$$

The term S_1 comes from the square of the abelian component of chromomagnetic field, the term S_2 from the chromoelectric field, while $S_3 + S_4 + S_5$ arise due to the square of the nonabelian part of the chromomagnetic field, the square of the term in eq. (3.7) which involves the covariant derivative of the Higgs field, and the interference term between the two. Defining

$$K = 1 - f\,, \tag{3.20}$$

we can rewrite

$$S_{\rm dandelion} = -\frac{\pi}{g^2}\ln(\Lambda L)\int_{-\infty}^{\infty} dz \int_{-1}^{1} dx \left\{[1-K^2-\theta(-z)]^2 + \left(\frac{\partial K}{\partial z}\right)^2 (1+2z^2) \right.$$
$$\left. + (1-x^2)\left(\frac{\partial K}{\partial x}\right)^2 + \theta(-z)\frac{1-x}{1+x}\left[K^2 + (1+x)\frac{\partial K^2}{\partial x}\right]\right\}, \qquad (3.21)$$

with Λ an ultraviolet cutoff, and L the linear size of the system.

The ultraviolet divergent has to do with the fact that our monopole has a zero core size, and is therefore not a problem. The infrared divergence appears since our ansatz for the dandelion vector potential is dilatationally invariant. As we will argue shortly, this is unavoidable and indicates that in perturbation theory the expectation value vanishes if the volume of space time is infinite. We believe this is an artifact of the perturbative calculation, and nonperturbative confinement effects must provide an effective infrared cutoff on this calculation, regulating this divergence.

The most important question is of course whether the remaining x and z integrals are finite. It is easy to see that this is the case for any smooth function which satisfies the following boundary conditions

$$\begin{aligned} &K(z,-1) = 0\,, \qquad z<0\,,\\ &|K(z\to\infty,x)-1| < \frac{1}{z}\,,\\ &|K(z\to-\infty,x)| < \frac{1}{z}\,. \end{aligned} \qquad (3.22)$$

We take as an ansatz for K functions of the form

$$K(z) = \rho(z)(1+x)^{1-\rho(z)}\,, \qquad (3.23)$$

where $\rho(z)$ is a smooth approximation to the step function. As an example consider the one parameter class of functions

$$\rho(z,\gamma) := \frac{1}{2} + \frac{2}{\pi}\arctan\left(\frac{\gamma z}{\sqrt{1+\gamma^2 z^2}}\right). \qquad (3.24)$$

From these we can construct a two-parameter class of functions $K(z,x,\alpha,\beta)$ where

$$K(z,x,\alpha,\beta) = \rho(z,\alpha)(1+x)^{1-\rho(z,\beta)}\,. \qquad (3.25)$$

Writing $S_{\rm dandelion}(\alpha,\beta) = -\frac{\pi}{g^2}\ln(\Lambda L)c(\alpha,\beta)$ and calculating the integrals numerically, we find as shown in Figure 1, a minimum value for the action at $\alpha \approx 1.35$ and $\beta \approx 2.65$ corresponding to $c(\alpha,\beta) \approx 2.17$. Several other smooth approximations to the step function have been investigated and all lead to a higher action.

A striking property of these boundary conditions is that a spherically symmetric ansatz, corresponding to the function K independent on the angle x has an action which diverges worse than logarithmically. In particular if we take

$$K = \sqrt{1 - \frac{1}{\pi}\left(\arctan\frac{1}{z} - \frac{z}{1+z^2}\right)}\,, \qquad (3.26)$$

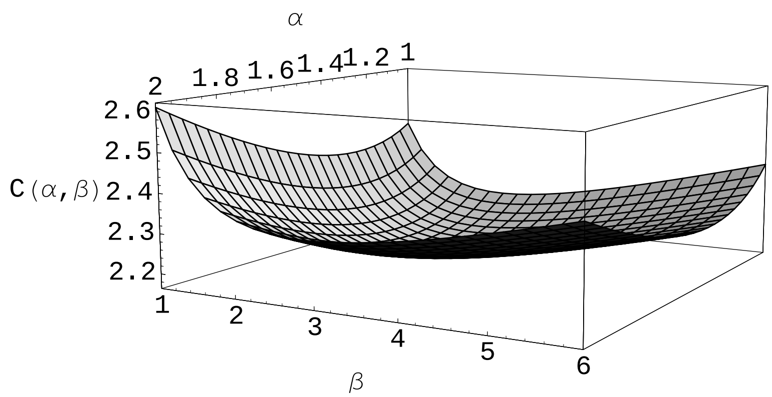

Figure 1: Plot of the two parameter variation $c(\alpha, \beta)$ for the dandelion action.

which gives the spherically symmetric magnetic current eq. (3.9), the z, x integral diverges as the second power of the infrared logarithm. The spherically symmetric configuration thus gives a negligible contribution to the path integral for the expectation value of M.

Physically the meaning of the dandelion configuration is quite clear. As $x_4 \to -\infty$, the magnetic field eq. (3.13) is the field of the point-like 'tHooft-Polyakov monopole. At negative but non-infinite times an extra magnetic charge density appears at distance scales of order x_4. This extra charge density is not spherically symmetric but rather has only axial symmetry and is pushed away from the Dirac string. Since our ansatz eq. (3.11) is dilatationally invariant, we expect that for any field of this type the extra magnetic charged density is small within the cone of the opening angle x_4/r around the string. As $x_4 \to \infty$, the field vanishes. Thus from this perspective it looks like a configuration that interpolates between a monopole in distant past and a vacuum in distant future. Note that even so, the conservation of the magnetic current cannot be violated by the dandelion. To see this explicitly we have to analyse the field profile at fixed x_4. As long as $r < |x_4|$, the field is what we have just described - a monopole at negative time, and vacuum at positive time. However at $r > |x_4|$, the field is given simply by the value of the function K at $z = 0$. As long as K is a continuous function of z, this means that the magnetic flux at spatial infinity is time independent. The balancing flux is precisely due to the extra magnetic charged density which lives at distances of order $r \propto x_4$. Thus the dandelion configuration describes at negative times a point-like 'tHooft-Polyakov monopole, which at radius $r \sim |x_4|$ is surrounded by an axially symmetric "shell" of negative magnetic charge density. As $|x_4| \to 0$ the radius of the shell shrinks to zero. At positive times the configuration has no core but instead all the magnetic charge density is concentrated in a shell whose size grows as $x_4 \to \infty$.

Note that the logarithmic infrared divergence is not an artifact of our dilatationally invariant ansatz. It is easy to see that one cannot do better than that. Let us relax the assumption that K depends only on the ration x_4/r. A little algebra then leads to the

following expression for the action of the dandelion configuration

$$
\begin{aligned}
S_{\text{dandelion}} = &-\frac{\pi}{g^2}\int dr \int_0^\infty dx_4 \int_{-1}^{1} dx \left[\frac{1}{r^2}(1-K^2)^2 + \left(\frac{\partial K}{\partial x_4}\right)^2 + 2\left(\frac{\partial K}{\partial r}\right)^2 \right.\\
&\qquad\qquad\qquad\qquad\qquad\qquad \left. + \frac{1}{r^2}(1-x^2)\left(\frac{\partial K}{\partial x}\right)^2\right] \qquad (3.27)\\
&-\frac{\pi}{g^2}\int dr \int_{-\infty}^{0} dx_4 \int_{-1}^{1} dx \left[\frac{1}{r^2}K^4 + \left(\frac{\partial K}{\partial x_4}\right)^2 + 2\left(\frac{\partial K}{\partial r}\right)^2 \right.\\
&\qquad\qquad\qquad\qquad\qquad\qquad \left. + \frac{1}{r^2}\frac{1-x}{1+x}\left(K + (1+x)\frac{\partial K}{\partial x}\right)^2\right].
\end{aligned}
$$

Let us follow the change of K with time $x_4 < 0$ at fixed value of r. The qualitative behaviour is clear from our discussion above. At large negative times $K = 0$, while at times close to zero, K is a fixed function $\kappa(x)$ which does not vanish at large r. Let's call the time interval during which this change in behaviour occurs $\Delta T(r)$. We can estimate the magnitude of various terms in eq. (3.27). In particular

$$
\begin{aligned}
&\int_{-\infty}^{0} dx_4 \frac{1}{r^2} K^4 \propto \frac{\Delta T(r)}{r^2}\kappa^4 , \qquad (3.28)\\
&\int_{-\infty}^{0} dx_4 \left(\frac{\partial K}{\partial x_4}\right)^2 \propto \frac{1}{\Delta T(r)}\kappa^2 .
\end{aligned}
$$

It is obvious from these two expressions that $T(r) \to_{r\to\infty} Ar$, or else the subsequent integral over r diverges stronger than logarithmically.

Thus we conclude that in the leading order in perturbation theory the expectation value of the monopole operator is

$$
\langle M\rangle = \exp\left(-\frac{\pi c}{g^2}\ln(\Lambda L)\right) \qquad (3.29)
$$

with constant $c \approx 2.17$.

4. Discussion

In this paper we have shown how to set up the perturbative calculation of the expectation value of the monopole creation operator in the confining phase. Perturbatively we find that the VEV vanishes in the infinite volume. However our result eq. (3.29) is an intrinsically perturbative one, and as such we certainly do not expect it to stand beyond perturbation theory. Nonperturbative contributions will provide an infrared cutoff. It is thus very likely that the $\langle M\rangle \neq 0$ in the confining phase.

A perturbative calculation of VEV is of course not reliable since it is dominated by the infrared physics. A quantity which is more amenable to such a calculation is short distance behaviour of the monopole-antimonopole correlation function [12]. Although we

have not calculated this quantity, the perturbative cutoff dependence of $\langle M \rangle$ suggests that the correlator behaves as

$$\left\langle M(x) M^{\dagger}(y) \right\rangle \propto |x-y|^{-\frac{\pi c}{g^2}} . \tag{4.1}$$

This indicates that even in the pure Yang-Mills limit the monopole operator does not freeze, even though its phase must decouple from the finite mass spectrum. Instead M couples to finite mass excitations via its modulus.

It is worth noting that the present approach can be easily extended to finite temperature. It is interesting to see whether at high temperature the VEV of M vanishes in perturbation theory.

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